1) a) Théorème de Thalès

$$\frac{AE}{ED} = \frac{EI}{EF} = \frac{AI}{DF}$$

$$\frac{x}{1+x} = \frac{EI}{EF} = \frac{AI}{1-x}$$

$$AI = \frac{x(1-x)}{1+x}$$

$$AI = \frac{x-x^2}{1+x}$$ pour $x \in]0;1[$

b) $u(x) = x - x^2$ $\quad u'(x) = -2x+1$

$v(x) = 1+x$ $\quad v'(x) = 1$

$$\left(\frac{u}{v}\right)' = \frac{(-2x+1)(1+x)-(x-x^2)1}{(1+x)^2}$$

$$\left(\frac{u}{v}\right)' = \frac{-2x-2x^2+1+\cancel{x}-\cancel{x}+x^2}{(1+x)^2}$$

$$\left(\frac{u}{v}\right)' = \frac{-x^2-2x+1}{(1+x)^2}$$

$-x^2-2x+1$

$= -[x^2+2x-1]$

$= -[(x+1)^2-2]$

$= -(x+1-\sqrt{2})(x+1+\sqrt{2})$

x	0		$\sqrt{2}-1$		1
$f'(x)$		$+$	0	$-$	
f		$\nearrow$	$3-2\sqrt{2}$	$\searrow$	

$AE = \sqrt{2}-1$ pour que AI soit maximale

2° $$\mathcal{A}(x) = \frac{AE \times AI}{2}$$

$$= \left(x \times \left(\frac{x-x^2}{1+x}\right)\right) \times \frac{1}{2}$$

$$= \left(\frac{x^2-x^3}{1+x}\right) \times \frac{1}{2}$$

$$= \frac{x^2-x^3}{2x+2}$$

$u(x) = x^2-x^3$ $\quad u'(x) = 2x-3x^2$

$v(x) = 2x+2$ $\quad v'(x) = 2$

$$\left(\frac{u}{v}\right)' = \frac{(2x-3x^2)(2x+2)-(x^2-x^3)2}{(2(x+1))^2}$$

$$\left(\frac{u}{v}\right)' = \frac{4x^2+4x-6x^3-6x^2-2x^2+2x^3}{(2(x+1))^2}$$

$$\left(\frac{u}{v}\right)' = \frac{-4x^3-4x^2+4x}{4(x+1)^2}$$

$$\left(\frac{u}{v}\right)' = \frac{-x^3-x^2+x}{(x+1)^2}$$

$$\left(\frac{u}{v}\right)' = \frac{-x(x^2+x-1)}{(x+1)^2}$$

$\Delta = 1^2 - 4 \times 1 \times (-1)$

$\Delta = 5$

$$x_1 = \frac{-1-\sqrt{5}}{2} \qquad x_2 = \frac{-1+\sqrt{5}}{2}$$

x	0		$\frac{-1+\sqrt{5}}{2}$		1
$\mathcal{A}'(x)$		$+$	0	$-$	
$\mathcal{A}$		$\nearrow$		$\searrow$	

$\mathcal{A}(x)$ est maximale pour $x = \frac{-1+\sqrt{5}}{2}$