```
> restart;with(linalg);
Warning, the protected names norm and trace have been redefined and
unprotected
```

[*BlockDiagonal*, *GramSchmidt*, *JordanBlock*, *LUdecomp*, *QRdecomp*, *Wronskian*, *addcol*, *addrow*, *adj*, *adjoint*, *angle*, *augment*, *backsub*, *band*, *basis*, *bezout*, *blockmatrix*, *charmat*, *charpoly*, *cholesky*, *col*, *coldim*, *colspace*, *colspan*, *companion*, *concat*, *cond*, *copyinto*, *crossprod*, *curl*, *definite*, *delcols*, *delrows*, *det*, *diag*, *diverge*, *dotprod*, *eigenvals*, *eigenvalues*, *eigenvectors*, *eigenvects*, *entermatrix*, *equal*, *exponential*, *extend*, *ffgausselim*, *fibonacci*, *forwardsub*, *frobenius*, *gausselim*, *gaussjord*, *geneqns*, *genmatrix*, *grad*, *hadamard*, *hermite*, *hessian*, *hilbert*, *htranspose*, *ihermite*, *indexfunc*, *innerprod*, *intbasis*, *inverse*, *ismith*, *issimilar*, *iszero*, *jacobian*, *jordan*, *kernel*, *laplacian*, *leastsqrs*, *linsolve*, *matadd*, *matrix*, *minor*, *minpoly*, *mulcol*, *mulrow*, *multiply*, *norm*, *normalize*, *nullspace*, *orthog*, *permanent*, *pivot*, *potential*, *randmatrix*, *randvector*, *rank*, *ratform*, *row*, *rowdim*, *rowspace*, *rowspan*, *rref*, *scalarmul*, *singularvals*, *smith*, *stackmatrix*, *submatrix*, *subvector*, *sumbasis*, *swapcol*, *swaprow*, *sylvester*, *toeplitz*, *trace*, *transpose*, *vandermonde*, *vecpotent*, *vectdim*, *vector*, *wronskian*]

```
> A:=matrix(8,8,[1,1,1,1,1,1,0,0,
                 1,0,0,0,0,0,0,0,
                 0,1,0,0,0,0,0,0,
                 8,8,8,8,8,8,0,0,
                 0,0,0,1,0,0,0,0,
                 0,0,0,0,1,0,0,0,
                 0,0,1,0,0,0,2,1,
                 0,0,0,0,0,1,8,9]);
```

$$A := \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 8 & 8 & 8 & 8 & 8 & 8 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 8 & 9 \end{bmatrix}$$

```
> U:=vector(8,[1,1,0,8,0,0,0,0]);
```

$$U := [1, 1, 0, 8, 0, 0, 0, 0]$$

factoriasation du polynôme caractéristique dans un corps de décomposition (on vire le facteur x^3 qui donnera un bloc de jordan nilpotent d'ordre 3

```
> P:=factor(charpoly(A,x));
> P2:=P/x^3;
>
  alias(alpha=RootOf(select(has,P2,x^3)));
  P2:=factor(P2,alpha);
```

```
alias(beta=RootOf(select(has,P2,x^2)));
P2:=factor(P2,beta);
P:=P2*x^3;
>
>
```

$$P := x^3\,(x-10)\,(x-1)\,(x^3-9\,x^2-9\,x-9)$$

$$P2 := (x-10)\,(x-1)\,(x^3-9\,x^2-9\,x-9)$$

$$\alpha, \beta$$

$$P2 := -(x^2-9\,x+x\,\alpha-9-9\,\alpha+\alpha^2)\,(x-1)\,(-x+\alpha)\,(x-10)$$

$$\alpha, \beta$$

$$P2 := (-x+\beta)\,(x-1)\,(-x+\alpha)\,(x-10)\,(x-9+\alpha+\beta)$$

$$P := (-x+\beta)\,(x-1)\,(-x+\alpha)\,(x-10)\,(x-9+\alpha+\beta)\,x^3$$

Calcul des matrices des projecteurs sur les différents espaces propres

```
> fac:=select(has,[op(P2)],x);
  for i in fac do
    Q[i]:=P/i;

  proj[i]:=evala(evalm(1/subs(x=solve(i,x),Q[i])*subs(x=A,Q[i])));

  od:
```

$$fac := [-x+\beta, x-1, -x+\alpha, x-10, x-9+\alpha+\beta]$$

```
>
> Apuisn:=evalm(add(vpn[i]*proj[i],i=fac)):
  res:=subs(seq(vpn[i]=solve(i,x)^(n-1),i=fac),collect((evalm(Apui
  sn&*U))[7],[seq(vpn[i],i=fac)]));
```

$$res := \left(\frac{1}{99}\,\alpha^2-\frac{59}{594}-\frac{1}{11}\,\alpha+\frac{1}{99}\,\beta\,\alpha-\frac{179}{1782}\,\beta\right)\beta^{(n-1)}+\frac{8}{9}+\left(-\frac{17}{1782}\,\alpha-\frac{1}{99}\,\alpha^2-\frac{5}{594}\right)\alpha^{(n-1)}$$
$$+\frac{10}{9}\,10^{(n-1)}+\left(-\frac{298}{297}+\frac{179}{1782}\,\alpha-\frac{1}{99}\,\beta\,\alpha+\frac{179}{1782}\,\beta\right)(-\beta+9-\alpha)^{(n-1)}$$

C'est le résultat formel en fonction de n , alpha et beta sont une racine quelconque respectivement des polynomes $x^3-9\,x^2-9\,x-9$

$x^2-9\,x+x\,\alpha-9-9\,\alpha+\alpha^2$

```
> evala(subs(n=11,res));
```

$$90971121$$

```
> time(evala(subs(n=1000000,res)));
>
>
```